A Mixed Integer Stochastic Optimization Model for Settlement Risk in Retail Electric Power Markets

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Abstract-- Recent changes in the U.S. electric power markets have contributed to volatility in hourly prices and loads. In this paper we consider the position of the electric power retailer who typically contracts with suppliers and end-users and must provide future load requirements to the suppliers. As part of this energy supply chain, the retailer is faced with great uncertainty in both market prices as well as end-user loads. Based on actual data for the PJM market covering Pennsylvania, New Jersey, and Maryland, we develop a probabilistic optimization model to optimize the net profits for the retailer for a forecast time horizon (typically one or more hours) given the cumulative performance in previous time periods (hours). The resulting model is formulated as a mixed integer linear program with binary variables due to the disjunctive nature of certain forward load estimation "bandwidth" tolerance constraints. In addition, we also provide an existence result to this optimization model. Lastly, we present a numerical example of the optimization model to validate its workings and provide some insight into model sensitivities.

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Index Terms—Stochastic Programming, Mixed Integer Programming, Decision-making, mathematical programming, risk analysis.

1. Introduction

The U.S. electric power markets are undergoing significant changes due to restructuring and deregulation and have experienced dramatic volatility in hourly prices. Accompanying these highly variable prices are fluctuating hourly loads. Thus, the job of determining both prices and loads for market participants has been especially difficult. Another complicating factor is the structural change of the century old industry from a vertically integrated monopoly into a non-integrated industry with numerous entities competing at various points in the supply chain network, which is a complicating factor. See (Conejo and Prieto, 2001) for an overview of the participants in this new marketplace.

Retail electricity deregulation has given us a new breed of market participants, the competitive "retailer", who fits into the supply chain between the wholesale supplier of electricity and the end-user. The role of the retailer is to act as an intermediary between the wholesalers and the end-users. The retailer purchases its supply from generators or traders, packages the product into smaller increments, and offers these packages to the end-user. Many types of products are offered by retailers including Fixed Price and Indexed products. This paper deals only with Fixed Price contracts.

The retailer faces very high risks when offering fixed price contracts because he has to manage some highly volatile components, such as customer demand, generation supply, transmission congestion, fuel costs, and customer retention. In addition, the margins available to the retailer are very limited during this transitional phase of moving from a regulated to a deregulated industry wherein the utility commissions have typically implemented price caps to protect the smaller customers from extreme price volatility.

The rest of this paper is organized as follows: Section 2 discusses the retailer's predicament; Section 3 contains a brief literature review; Section 4 describes the nomenclature and key elements of the model; Section 5 presents the model formulation and theoretical results concerning existence of a solution to this model; Section 6 describes sample results when using the model, and Section 7 summarizes the conclusions.

2. The Retailer's Predicament

The retailer has both a set of suppliers [4] who supply electricity to them and a set of end-users who are served by the retailer. The retailer has in effect two sets of contracts to manage – one with the suppliers, and the other with the end-users. The retailer's intent is that the revenues generated from the end-users will cover the costs of goods to be paid to the supplier, leaving a decent margin for the retailer. A depiction of this part of the energy supply chain network is given in Figure 1.

[4] "Suppliers" are taken to mean other market participants such as generators, power marketers and power traders.

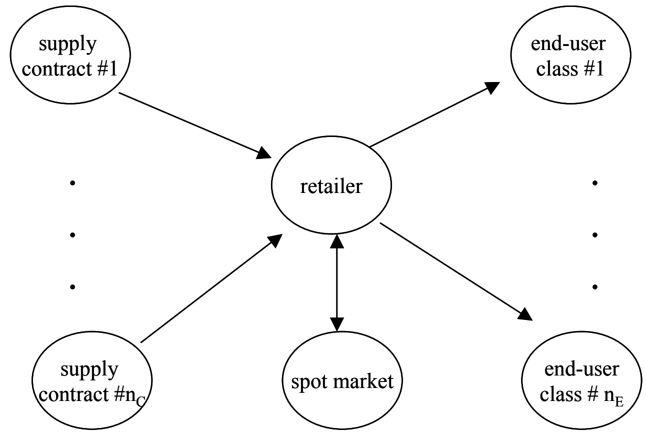

Fig. 1. The Two Step Distribution System Being Modeled.

It is often the case that the retailer must provide some sort of future load position to the suppliers as part of the contractual arrangements. Also, the contracts between the end-users and the retailer can stipulate that for a fixed price, the retailer must meet the load requirements of the end-users at whatever level they may be. In the event of over-estimating, the retailer can sell the excess electricity to the spot market at the prevailing spot market price. Conversely, if the retailer has under-estimated the load requirements of the end-user, deficient electricity can be purchased from the spot market at the associated spot market price. An interesting point to note is that if the retailer is interested in maximizing net profit, it is not necessarily advisable to try to determine their forward load positions to the supplier as accurately as possible as was shown in (Gabriel *et al.*, 2002). Indeed, as shown in Table 1, there are six cases that must be analyzed when considering over- or under-estimating in conjunction with the relative positions of the spot market price, the end-user price, and the supplier price, given respectively for hour h as $\mathrm{PRICE}_h^{SM}, \mathrm{PRICE}_h^{EU}, \mathrm{PRICE}_h^{SU}$. [5]

TABLE 1

The Various Results When Considering Over- and Under-Estimating the Load.

PRICES	LOAD ESTIMATE	RESULT
1. $PRICE_h^{SU} < PRICE_h^{EU} < PRICE_h^{SM}$	Over	Profit
2. $PRICE_h^{SU} < PRICE_h^{EU} < PRICE_h^{SM}$	Under	Loss
3. $PRICE_h^{SU} < PRICE_h^{SM} < PRICE_h^{EU}$	Over	Profit
4. $PRICE_h^{SU} < PRICE_h^{SM} < PRICE_h^{EU}$	Under	Profit
5. $PRICE_h^{SM} < PRICE_h^{SU} < PRICE_h^{EU}$	Over	Loss
6. $PRICE_h^{SM} < PRICE_h^{SU} < PRICE_h^{EU}$	Under	Profit

In terms of additional supply markets, for the purposes of this paper, we assume that the retailer is not concerned with the day- or hour-ahead markets but can only receive or sell electricity to the spot market. This is not an unreasonable assumption in practice as some retailers assign the task of transacting with the day-ahead and hour-ahead markets to the wholesalers or suppliers with whom they hold contracts; actual market rules can however vary by jurisdiction. In the same vein, we do not consider the retailer using financial options or other instruments (Hull, 2000), which is also consistent with some retailers’ strategies. [6]

[5] This set of cases makes the realistic assumption that the end-user price has been set higher than the supplier price.
[6] Private communications with a retailer in the northeastern U.S.

The focus of this paper is a stochastic optimization model that will assist power retailers in determining their optimal forward load positions vis-à-vis their suppliers and end-users. Such a model necessarily involves the notion of "settlement risk" which can be defined as the uncertainty in the total price paid for power supply by the retailer because of uncertainty in the final delivered power in each hour and the uncertainty in the marginal market clearing price during that hour. This settlement risk is an important and is discussed in detail in a later section.

3. Literature Review

Not much work in this aspect of retail electric power markets has appeared in the literature to date with the exception of (Gabriel *et al*., 2002) based in part on (Genc, 2001). These works evaluated heuristic strategies for determining forward load positions and via simulation, established which ones were best in terms of net profit. The results from these works showed, for example, that if the retailer always took the largest possible forward position for load, this strategy would stochastically dominate [7] all the others considered for the market in question. Thus, providing an accurate forward position to the supplier was not important in spite of certain forward load estimation "bandwidth" tolerance constraints.[8] This result is not immediate when one considers the six possible cases shown in Table 1 and the set of constraints described in (Gabriel *et al*., 2002) and the fact that just a few hours with, for example, very high or very low spot market prices can greatly influence the results and produce very large positive or negative settlement profits. As such the current work represents an extension of this paper. It

[7] See (Clemen and Reilly, 2001) for a discussion of stochastic dominance.

[8] "Bandwidth" here refers not to communications capacity (e.g., a high-bandwidth Internet connection) but rather to a "band" around the load estimate for which the retailer is allowed to deviate without assuming more of the settlement risk.

should be noted that these results may differ by market however, it is a function of the contracts, random loads and prices.

The current work and that of (Gabriel *et al.* 2002) should be contrasted with somewhat limited literature on modeling electric power retailers using stochastic programming. For the most part, other retail electric power papers have been geared more toward descriptive aspects of load estimation such as statistical modeling of loads. By contrast, the current work differs in that it is prescriptive, what the retailer should do, rather than descriptive in terms of future loads. The current work also differs from (Gabriel *et al.* 2002) substantially since optimal solutions are obtained by solving a certain mixed integer linear program in the current work as opposed to using simulation as a analysis tool.

There have been numerous papers involving stochastic programming in electricity or more generally energy but they have focused on other aspects of the energy supply chain network besides the retailer. Part of the reason for this is the recent arrival of this participant into the marketplace. The recent survey paper by Wallace and Fleten (2002) describes many of these previous stochastic programming works in energy. A brief selection of examples of stochastic modeling work in energy includes the following: electric power generation (Takriti *et al.,* 1996; Hobbs and Ji, 1999; Pérez-Ruiz and Conejo, 2000), capacity expansion (Sherali *et al.,* 1984), bilateral trading analyses (Bower and Bunn, 2000), environmental aspects of energy (Kanudia and Loulou, 1998), and natural gas planning (Haurie *et al.,* 1987; DeWolf and Smeers, 1997; Gürkan *et al.,* 1999). The current work does share some similarity with these papers in that energy decisions are to be made in the face of uncertainty.

Also important to note, uncertain prices and loads are described in the current paper by empirically-based, conditional probability distributions established from real data for the PJM market (Gabriel *et al.*, 2002). These distributions allow for auto-correlation between values in consecutive hours in that the current hour's distribution (for price or load) is one of three possible ones depending on the previous hour's value (low, medium or high) based on the notion of a one-step probability transition matrix consistent with random walks. Separate distributions were calibrated for each peak/offpeak designation and season, resulting in 24 separate distributions for price and 24 for load. This approach is believed to be flexible and realistic but differs from other approaches which capture the auto-correlation in prices or loads with other mechanisms for example, based on stochastic processes.

In this work, theoretical results concerning the existence of a solution to the model are presented and a small numerical example with a sensitivity analysis is used to illustrate the workings of the model. The conclusions from this small model would not necessarily apply to every market as the relevant data and assumptions may differ. This was the case in a related study applied to the Texas (ERCOT) market (Gabriel *et al.*, 2003) whose conclusions did not entirely match what was found in this paper and (Gabriel *et al.*, 2002) for the Pennsylvania, New Jersey, Maryland (PJM) market. However, the theoretical results as well as the general framework remain valid for different markets. Also, such a model if used by an actual retailer would necessarily need to be computationally enhanced for a longer time frame considered. Computational efficiency is not considered here but rather a presentation and validation of the model in question.

4. Nomenclature and Explanation of Key Model Elements

In this section we describe notational aspects of the model and give an example of the important bandwidth tolerance constraints. We note that the model to be presented will typically operate at the hourly level but that the time index "*h*" could refer to an hour, a day, a week, etc. as appropriate. At a set of target hours (times) $\widetilde{\mathrm{H}}$, the retailer evaluates his cumulative financial performance. These times might correspond for example to the end of the quarter during which important business decisions are made; the typical target hour in this set is designated as $\tilde{h}$.

Before the first hour h_I that the optimization model is run, the retailer has some historical data related to the financial performance of his operations. These data typically will be the cumulative profit (e.g., for the year or quarter) up to that point. From h_I on, the optimization model is run to determine the best estimation strategy for the retailer relative to their supply contracts. The figure $TOTPROF_{h_I-1}$ represents the total cumulative net profit at hour h_I-1, i.e., before the model runs. The full set of hours in which the optimization model is to be run are designated as $\overline{\mathrm{H}}$. To clarify the notions of the different time periods, consider the following time axis.

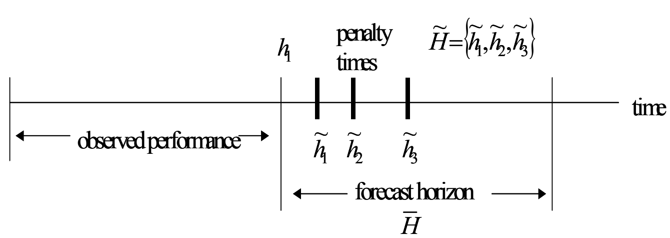

Fig. 2. Sample Timeframe.

The observed performance of the retailer $TOTPROF_{h_1-1}$, occurs outside of the model timeframe and is thus exogenous to the model. After this time, there are selected time periods , $\tilde{H} = \{\tilde{h}_1, \tilde{h}_2, \tilde{h}_3\}$ at which cumulative net profit goals are enforced.

The retailer interacts with suppliers via contracts that have previously been set up. The data and indices referring to these contracts are as follows:

c = Index for supply contracts

C = Set of contracts ,

where $PRICE^{SU}_{c,h}$ is the contractual supplier price, expressed in \$/MWh. [9] In terms of end-users, we model a set of end-user classes E with a typical class given by the index e. These classes are a collection of consumers contractually linked to the retailer with each class having:

1. At least one consumer and

2. All consumers within an end-user class exhibiting similar load patterns.

The number of end-users within a particular class $e \in E$ is given by N_e . For this class, the retailer has a contract (or contracts) that stipulates a price, $PRICE^{EU}_{e,h}$ expressed in \$/MWh, at which the end-users will purchase their electricity. Furthermore, we designate the set of end-user classes served by supply contract c as $E(c)$.

We make the following reasonable assumptions related to end-user classes.

Assumption 1:

Each end-user class is served by some contract, i.e., $E = \bigcup_{c \in C} E(c)$.

Assumption 2 :

An end-user class is served by just one contract, i.e., $E(c) \cap E(c') = \varnothing$ for $c \neq c'$.

Additionally, the end-users are considered as one aggregated group in this study and not broken down into different sectors (residential, commercial, industrial) or individual customer accounts.

As part of the retailer's decision-making, the uncertain end-user load must be taken into account. We denote the random end-user load that the retailer must satisfy by $LOAD_{e,h}^{A}(\omega_L)$ where $\omega_L \in \Omega_L$ is the index for a certain realization from the finite set of possible loads Ω_L. [10] Associated with this random load is a marginal probability mass function $\overline{\pi}_{e,h}(\omega_L)$ that describes the chance of realizing this particular load.

Another random variable is the nonnegative [11] spot market price $PRICE_h^{SM}(\omega_P)$ indexed by the realization $\omega_P \in \Omega_P$, Ω_P being the finite set possible realization indices. Both the spot market price, expressed in \$/MWh, as well as the random end-user load, are indexed by $\omega = (\omega_L, \omega_P) \in \Omega$ with $\Omega = \Omega_L \times \Omega_P$ [12] with associated joint probability mass function given as $\pi_{e,h}(\omega)$.

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[9] MW= megawatt, MWh = megawatt hour.
[10] Expressed in MW.
[11] Negative prices have been observed in some markets and can presumably reflect the preference of generators to keep their machines operating rather than stopping and later restarting them at a substantial cost. In this paper, based on the PJM data we obtained, no negative prices were observed and are thus not considered in our model.
[12] A finite set of possible end-user loads and spot market prices has been chosen for ease of presentation and without loss of realism although an infinite set of values could also have been selected.

Within this setting of random end-user loads and spot market prices, the retailer must decide on the particular forward load estimates to make to the suppliers. For supply contract c, end-user class e, this nonnegative estimate, expressed in MW, is denoted as $LOAD^{F}_{c,e,h}$ which is bounded above by the maximum allowable load forecast $M_{c,h}$.

One of the key concepts in the retailer-supplier relationship is the notion of forward load estimation "bandwidth" tolerance constraints. These constraints, between the supplier and the retailer reward the retailer when the estimated load to the supplier is within a certain tolerance (typically around 8%), of what actually was the end-user load; consider the following example. Suppose that for a particular hour, the retailer has estimated that the load to one of its end-user classes will be 100 MW. This forecast is then used by the supplier in terms of providing the appropriate amount of electricity. If however, the actual load for this end-user class was only 80 MW, then the retailer will have over-estimated by 20%. With a tolerance of 8%, the retailer would find that its forecast was outside the bandwidth tolerance and whatever spot market revenues or costs were associated with this over-forecast should be borne by itself (or at least a large share of it). Note that it is not always the case that over-estimating is good (see cases 1,3, and 5 in Table 1). If however the actual load were closer to this 100 MW, say 105MW resulting in under-estimating by 5% (within the 8% tolerance), then the retailer and the supplier would more nearly split the associated settlement revenues (from needing to transact with the spot market to buy or sell electricity).

In all, we distinguish three regions of interest for these bandwidth tolerance constraints: when

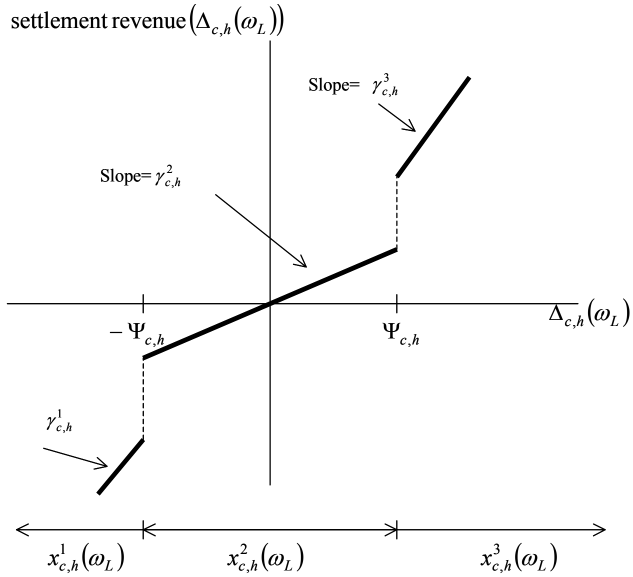

Fig. 3. Depiction of the Bandwidth Tolerance Constraints and Settlement Revenue.

the difference between the forecasted load and the actual load is negative and outside the tolerance (to the far left in Figure 3), when this difference is positive and outside the tolerance

(to the far right in Figure 3) and when this difference is within the tolerance (the middle segment in Figure 3).

To this end we define the following intermediate variables.

$\Delta_{c,h}(\omega_L) \quad = \sum_{e \in E(c)} N_e \left(LOAD^F_{c,e,h} - LOAD^A_{e,h}(\omega_L) \right)$ which is the difference between the total forecasted and total actual load made to supplier c and

$\Psi_{c,h} = T_{c,h} \sum_{e \in E(c)} N_e LOAD^F_{c,e,h}$ which is the percentage cutoff amount in MW as specified by the bandwidth cutoff % $T_{c,h}$. For example, in the case of over-estimating as described above, we'd have (with $E(c) = \{e\}, N_e = 1, T_{c,h} = 0.08$)

$\Delta_{c,h}(\omega_L) = 100 - 80 = 20$ (MW) and

$\Psi_{c,h} = 0.08(100) = 8$ MW.

The three tolerance constants $\gamma^1_{c,h}, \gamma^2_{c,h}, \gamma^3_{c,h}$ are the slopes of the function shown in Figure 3 and typically one might have $\gamma^1_{c,h} = \gamma^3_{c,h} = 1.0$, , i.e., the retailer having poorly forecasted the load and taking the full settlement amount (positive or negative) and $\gamma^2_{c,h} = 0.5$, corresponding to the retailer making an accurate forecast and splitting the settlement revenue evenly with the supplier. Lastly, the variables $x^1_{c,h}(\omega_L)$, $x^2_{c,h}(\omega_L), x^3_{c,h}(\omega_L)$ are the amounts, at most one of which is nonzero, correspdoning to the load forecast less the actual load for each segment 1,2, and 3, respectively, in Figure 3. These variables are defined as follows:

$$\Delta_{c,h}(\omega_L) = x^1_{c,h}(\omega_L) + x^2_{c,h}(\omega_L) + x^3_{c,h}(\omega_L).$$

Another part of the model involves retailer financial goals. We assume that the retailer has decided ahead of time on a target set of net profits to be met over time. [13] Relative to the financial well-being of the retailer, there are a target set of cumulative net profit goals that must be met at certain key time intervals indexed by the set $\tilde{H}$ described above. There is some allowance for deviations below these target levels but a lower acceptable limit is also stipulated. This limit represents the absolute lowest amount of cumulative net profits that can be supported by the marketer before a penalty is incurred (e.g., needing to get additional financing from somewhere). Since the cumulative net profits are themselves random variables, being a function of the stochastic prices and loads, we translate the above requirements to be the following for each target hour $\tilde{h} \in \tilde{H}$:

For $\tilde{h} \in \tilde{H}$, this constraint is given as:

$$\eta_{\tilde{h}}^{min} \leq \text{cumulative net profit at hour } \tilde{h}$$

where:

$\eta_{\tilde{h}}^{\min}$ = the minimum acceptable cumulative net profit at hour $\tilde{h} \in \tilde{H}$

We want to force a penalty as discussed above if the cumulative net profit at hour $\tilde{h}$ drops below $\eta_{\tilde{h}}^{\min}$. Thus, we want

$$penalty_{\tilde{h}} = \rho \, max\left\{\eta_{\tilde{h}}^{\min} - \text{cumulative net profit at hour } \tilde{h}, 0\right\}$$

where:

ρ = The positive penalty percentage (e.g., interest rate).

Figure 4 below depicts these financial aspects described above.

[13] We assume that the set-up costs for contracts are excluded. These constraints are for operational goals only.

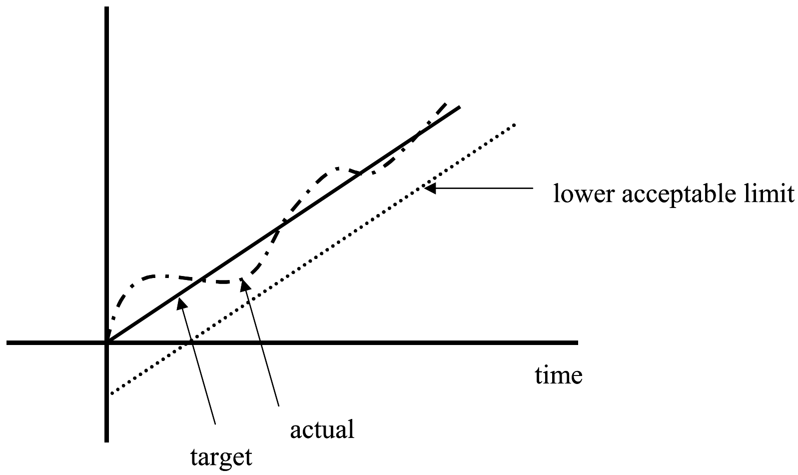

Fig. 4. Target Cumulative Net Profit Over a Period of Time.

5. Model Formulation

Two common approaches for optimizing in the presence of uncertainty are to use either recourse or chance constraints (Birge and Louveax, 1997); our model is of the recourse variety. With the recourse approach, a two-stage optimization is performed with the first stage relating to initial decisions that must be made before uncertain elements are resolved. Examples include investment decisions or more generally planning decisions. In our setting, these first-stage variables are the forward load estimates that the retailer provides to the supplier before observing the actual (random) end-user loads.

In the second stage, for a particular realization of the uncertain items (e.g., end-user load), one solves another optimization problem with the first-level planning decisions being fixed. In the current settings, the second-stage variables are how much electricity to be bought from or sold to the spot market. These two stages can be combined in two ways. The first way to combine them uses the "extensive form" where all scenarios are spelled out, resulting in a much larger but highly structured optimization problem. The second way is to combine them implicitly by adding to the first-stage objective function an expected recourse function representing the expected optimal value of the second-stage optimization (Birge and Louveax, 1997). We have chosen to use the extensive form for clarity of presentation since our intent is to describe the model formulation rather than exploring algorithmic aspects for solving this model.

In what follows, we present a mixed integer programming model to solve for the retailer's forward load positions and describe each part below noting that $\varepsilon > 0$ is a suitably small constant and $M > 0$ is a suitably large constant. Typically, a retailer may wish to solve this optimization problem for entire forecast horizon (many hours) as depicted in Figure 2. Alternatively, such a model could also be solved repeatedly for each set of hours with the observed performance updated on some regular basis.

Maximize

(Expected end-user revenues)

$$\left\{ \sum_{h \in \bar{H}} \sum_{\omega_L \in \Omega_L} \sum_{c \in C} \sum_{e \in E(c)} \left(PRICE_{e,h}^{EU} \right) N_e LOAD_{e,h}^{A}(\omega_L) \bar{\pi}_{e,h}(\omega_L) + \right.$$

(Expected settlement revenues)

$$+ \sum_{h \in \bar{H}} \sum_{\omega \in \Omega} \sum_{c \in C} \sum_{e \in E(c)} \left\{ \gamma_{c,h}^{1} PRICE_{h}^{SM}(\omega_P) \pi_{e,h}(\omega) x_{c,h}^{1}(\omega_L) + \right.$$
$$\left. \gamma_{c,h}^{2} PRICE_{h}^{SM}(\omega_P) \pi_{e,h}(\omega) x_{c,h}^{2}(\omega_L) + \gamma_{c,h}^{3} PRICE_{h}^{SM}(\omega_P) \pi_{e,h}(\omega) x_{c,h}^{3}(\omega_L) \right\}$$

(Supply costs)

$$-\sum_{h\in\overline{H}}\sum_{c\in C}\sum_{e\in E(c)} PRICE^{SU}_{c,h}\; N_e \sum_{e\in E(c)} LOAD^{F}_{c,e,h}$$

(Cumulative net profit penalty costs)

$$-\sum_{\tilde{h}\in\tilde{H}}\rho\left(penalty_{\tilde{h}}\right)\Bigg\} \quad (1a)$$

subject to

(Definitional constraints)

$$\Delta_{c,h}(\omega_L)=\sum_{e\in E(c)} N_e\left(LOAD^{F}_{c,e,h}-LOAD^{A}_{e,h}(\omega_L)\right) \quad (1b)$$

$$\Psi_{c,h}=T_{c,h}\sum_{e\in E(c)} N_e LOAD^{F}_{c,e,h} \quad (1c)$$

(Tolerance constraints) $\forall c\in C, h\in\overline{H}, \omega_L\in\Omega_L$

$$\Delta_{c,h}(\omega_L)=x^{1}_{c,h}(\omega_L)+x^{2}_{c,h}(\omega_L)+x^{3}_{c,h}(\omega_L) \quad (1d)$$

$$-My^{1}_{c,h}(\omega_L)\le x^{1}_{c,h}(\omega_L)\le 0 \quad (1e)$$

$$x^{1}_{c,h}(\omega_L)+\Psi_{c,h}+\varepsilon\le M\left(1-y^{1}_{c,h}(\omega_L)\right) \quad (1f)$$

$$-\Psi_{c,h}\le x^{2}_{c,h}(\omega_L)\le\Psi_{c,h} \quad (1g)$$

$$-My^{2}_{c,h}(\omega_L)\le x^{2}_{c,h}(\omega_L)\le My^{2}_{c,h}(\omega_L) \quad (1h)$$

$$0\le x^{3}_{c,h}(\omega_L)\le My^{3}_{c,h}(\omega_L) \quad (1i)$$

$$-x^{3}_{c,h}(\omega_L)+\Psi_{c,h}+\varepsilon\le M\left(1-y^{3}_{c,h}(\omega_L)\right) \quad (1j)$$

$$y^{1}_{c,h}(\omega_L)+y^{2}_{c,h}(\omega_L)+y^{3}_{c,h}(\omega_L)=1 \quad (1k)$$

$$y^{1}_{c,h}(\omega_L),\, y^{2}_{c,h}(\omega_L),\, y^{3}_{c,h}(\omega_L)\in\{0,1\} \quad (1l)$$

(Cumulative net profit constraints) $\forall \tilde{h} \in \tilde{H}, \omega \in \Omega$

$$\begin{aligned} penalty_{\tilde{h}} \geq\ & \eta_{\tilde{h}}^{min} \\ & -\Big[TOTPROF_{h_1 - 1} \\ & + \sum_{h_1 \leq h \leq \tilde{h}} \sum_{c \in C} \sum_{e \in E(c)} PRICE_{e,h}^{EU} N_e LOAD_{e,h}^{A}(\omega_L) + \sum_{h_1 \leq h \leq \tilde{h}} \sum_{c \in C} \Big\{\gamma_{c,h}^{1} PRICE_h^{SM}(\omega_P) x_{c,h}^{1}(\omega_L) + \\ & \gamma_{c,h}^{2} PRICE_h^{SM}(\omega_P) x_{c,h}^{2}(\omega_L) + \gamma_{c,h}^{3} PRICE_h^{SM}(\omega_P) x_{c,h}^{3}(\omega_L)\Big\} \\ & - \sum_{h_1 \leq h \leq \tilde{h}} \sum_{c \in C} \sum_{e \in E(c)} PRICE_{c,h}^{SU} N_e LOAD_{c,e,h}^{F}\Big] \quad \forall \tilde{h} \in \tilde{H} \end{aligned} \tag{1m}$$

$$penalty_{\tilde{h}} \geq 0 \quad \forall \tilde{h} \in \tilde{H} \tag{1n}$$

(Forward Load Bounds)

$$0 \leq \mathrm{LOAD}_{c,e,h}^{F} \tag{1o}$$

$$\mathrm{LOAD}_{c,e,h}^{F} \leq \mathrm{M}_{c,h} \tag{1p}$$

A. Objective Function

The objective function is the sum of expected revenues from selling the electricity to the end-users, expected settlement revenues via the spot market (positive or negative), less the supply costs of the estimated load and the penalties incurred by not meeting the minimum acceptable cumulative net profit targets.

The end-user revenues are determined by multiplying the end-user price for end-user class e and hour h, $PRICE_{e,h}^{EU}$, by the number of end-users in the class N_e, and the actual load $LOAD_{e,h}^{A}(\omega_L)$ for the realization ω_L of the load. This gives the total dollars gained from that end-user class in that hour. This term is then multiplied by the probability of that load realization $\overline{\pi}_{e,h}(\omega_L)$ and summed over the appropriate indices to arrive at total expected end-user revenues. Note that this term does not involve any decision variables and is thus a constant. Rather than dropping it from the formulation, it has been shown here for completeness.

The expected settlement revenues term is computed by multiplying the spot market price for hour h, $PRICE_h^{SM}(\omega_P)$ associated with the price realization ω_P by the estimate of the difference between forward load and actual load for the realization of the load and price given by $\omega = (\omega_L, \omega_P)$. This difference given by $x_{c,h}^i(\omega_L)$ ($i = 1,2,3$) corresponds to the values on the horizontal axis in Figure 3. As described below, exactly one of the three terms in this part of the objective function will be nonzero. Each of the terms is then multiplied by the appropriate values $\gamma_{c,h}^i$ ($i = 1,2,3$), which determine the share of the settlement revenue with contractor c for hour h that goes to the retailer. All these terms are then summed over the appropriate indices to arrive at the expected settlement revenues that the retailer will obtain.

The remaining two parts of the objective function concern the supply costs and the penalty costs. The former is simply the total sum of estimated forward load multiplied by the supplier contract price and the latter is the penalty interest rate ρ multiplied by the amount of the penalty $penalty_{\tilde{h}}$ obtained from (1m) and (1n) and summed over the appropriate indices.

B. Constraints

The optimization problem (1) is subject to several sets of constraints that enforce realistic contractual aspects as well as consistency. In what follows we describe these constraints.

1) <u>*Bandwidth Tolerance*</u> The retailer typically has a part of the contract that specifies penalties if their load estimates do not match actual ones by a certain amount. For a realization ω_L of the load, these bandwidth tolerance constraints are of the following form.

$$\text{If } \left| \frac{\sum_{e \in E(c)} N_e \left(LOAD^F_{c,e,h} - LOAD^A_{e,h}(\omega_L) \right)}{\sum_{e \in E(c)} N_e LOAD^F_{c,e,h}} \right| \le T_{c,h} \tag{2}$$

then the retailer earns

$$\gamma^2_{c,h} \sum_{e \in E(c)} PRICE^{SM}_h(\omega_P) N_e \left(LOAD^F_{c,e,h} - LOAD^A_{e,h}(\omega_L) \right)$$

where $\gamma^2_{c,h} \in (0,1)$ and is typically around 0.5. That is, the retailer and the supplier for contract *c* evenly split the positive or negative spot market revenues for the end-users served by the contract *c*. If the retailer has over-estimated and is beyond the bandwidth tolerance percentage, the retailer earns

$$\gamma^3_{c,h} \sum_{e \in E(c)} PRICE^{SM}_h(\omega_P) N_e \left(LOAD^F_{c,e,h} - LOAD^A_{e,h}(\omega_L) \right)$$

where $\gamma^3_{c,h} \in (0,1)$ and is typically around 1.00. When the retailer has under-estimated and beyond the bandwidth tolerance percentage the retailer earns

$$\gamma^1_{c,h} \sum_{e \in E(c)} PRICE^{SM}_h(\omega_P) N_e \left(LOAD^F_{c,e,h} - LOAD^A_{e,h}(\omega_L) \right)$$

where $\gamma_{c,h}^{1} \in (0,1)$ and is typically around 1.00 as well. In other words, the retailer takes the major amount of the positive or negative spot market revenues when outside this bandwidth tolerance. [14] The disjunctive nature of such contracts makes the analysis not straightforward and can be clearly seen in Figure 3 for a typical contract and hour.

We note that the purpose of constraint (1d) is to break up the difference between forward load estimate and actual load into three pieces described by the three variables: $x_{c,h}^{1}(\omega_L)$, $x_{c,h}^{2}(\omega_L)$, $x_{c,h}^{3}(\omega_L)$ at most one of which is nonzero, consistent with Figure 3. We have the following result describing the logic in contraints (1b)-(1p) insuring the consistency of the variables associated with Figure 3.

Theorem 1

With M a suitably large, positive constant, constraints (1b)-(1p) ensure the following:

i. $\begin{cases} x_{c,h}^{1}(\omega_L) < 0 \Rightarrow x_{c,h}^{1}(\omega_L) \in [-M, -\Psi_{c,h}) \\ x_{c,h}^{1}(\omega_L) = 0, otherwise \end{cases}$

ii. $\begin{cases} x_{c,h}^{2}(\omega_L) \neq 0 \Rightarrow x_{c,h}^{2}(\omega_L) \in [-\Psi_{c,h}, \Psi_{c,h}] \\ x_{c,h}^{2}(\omega_L) = 0, otherwise \end{cases}$

iii. $\begin{cases} x_{c,h}^{3}(\omega_L) > 0 \Rightarrow x_{c,h}^{3}(\omega_L) \in (\Psi_{c,h}, M] \\ x_{c,h}^{3}(\omega_L) = 0, otherwise \end{cases}$

At most one of the variables $x_{c,h}^{1}(\omega_L)$, $x_{c,h}^{2}(\omega_L)$, $x_{c,h}^{3}(\omega_L)$ is nonzero.

Proof

[14] Allowing three bandwidth constants as opposed to just two for inside or outside the bandwidth is a slight

First note by (1e) that $x^1_{c,h}(\omega_L) \le 0$ and that if $x^1_{c,h}(\omega_L) < 0 \Rightarrow y^1_{c,h}(\omega_L) = 1$, otherwise there is a contradiction with (1e). From (1e) we see the lower bound of $-M$. We note that $y^1_{c,h}(\omega_L) = 1$ means that in (1f), $x^1_{c,h}(\omega_L) + \varepsilon \le -\Psi_{c,h}$ which since $\varepsilon > 0$ implies that $x^1_{c,h}(\omega_L) < -\Psi_{c,h}$. The other case is that $x^1_{c,h}(\omega_L) = 0$ which implies by (1f) that $y^1_{c,h}(\omega_L) = 0$ since if not, then in light of the nonnegative load estimates, we'd have $0 < \Psi_{c,h} + \varepsilon \le M\left(1 - y^1_{c,h}(\omega_L)\right) = 0$, a contradiction. In particular, these constraints show that $x^1_{c,h}(\omega_L) < 0 \Rightarrow y^1_{c,h}(\omega_L) = 1$, $x^1_{c,h}(\omega_L) = 0 \Rightarrow y^1_{c,h}(\omega_L) = 0$.

Constraints (1g) and (1h) provide the logic for the variable $x^2_{c,h}(\omega_L)$. In particular, we see that when $x^2_{c,h}(\omega_L) \ne 0 \Rightarrow y^2_{c,h}(\omega_L) = 1$, otherwise there would be a contradiction with (1h). Since $M > 0$ was chosen suitably large, we see that the bounds in(1g) are tighter than those in (1h) so that $x^2_{c,h}(\omega_L) \ne 0 \Rightarrow x^2_{c,h}(\omega_L) \in \left[-\Psi_{c,h}, \Psi_{c,h}\right]$. In particular, these constraints show that $x^2_{c,h}(\omega_L) \ne 0 \Rightarrow y^2_{c,h}(\omega_L) = 1$. If $x^2_{c,h}(\omega_L) = 0$, both $y^2_{c,h}(\omega_L) = 0 \text{ or } 1$ is a solution.

From (1i), we see that when $x^3_{c,h}(\omega_L) > 0 \Rightarrow y^3_{c,h}(\omega_L) = 1$, otherwise there would be a contradiction. This implies via (1i) that $x^3_{c,h}(\omega_L) \le M$ and via (1j) that $\Psi_{c,h} + \varepsilon \le x^3_{c,h}(\omega_L)$ or that $\Psi_{c,h} < x^3_{c,h}(\omega_L)$. When, $x^3_{c,h}(\omega_L) = 0 \Rightarrow y^3_{c,h}(\omega_L) = 0$, since otherwise in (1j) we'd have $0 < \Psi_{c,h} + \varepsilon \le M\left(1 - y^3_{c,h}(\omega_L)\right) = 0$, a contradiction.

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generalization to what was presented in (Gabriel *et al.*, 2002).

Lastly, constraint (1k) guarantees that at most one of variables $x^1_{c,h}(\omega_L)$, $x^2_{c,h}(\omega_L)$, $x^3_{c,h}(\omega_L)$ is nonzero. ■

Remarks:

1. Note that the fact that $y^2_{c,h}(\omega_L)=0 \text{ or } 1$ when $x^2_{c,h}(\omega_L)=0$ is not a problem. This follows since if $x^2_{c,h}(\omega_L)=0$ but $x^1_{c,h}(\omega_L)\neq 0$ or $x^3_{c,h}(\omega_L)\neq 0$, so that either $y^1_{c,h}(\omega_L)=1$ or $y^3_{c,h}(\omega_L)=1$, by constraint (1k) we see that $y^2_{c,h}(\omega_L)=0$. Also, when all three *x* variables =0, that is, the forward load estimation matches exactly the actual load, it is possible that $y^2_{c,h}(\omega_L)=1$. In this case, there is no harm in having this particular *y* value since the logic of the constraints is preserved with no adverse effects on the objective function.

2. The conditions i., ii., and iii., in Theorem 1 correspond respectively to: under-estimating the load and being outside the bandwidth, within the bandwidth tolerance, and over-estimating and outside the bandwidth. The logic of these conditions insures that the settlement revenue is computed correctly as described above.

<u>2) *Cumulative Net Profits* </u>The constraints (1m) and (1n) establish that the maximum possible penalty to be paid under all scenarios, i.e., $\forall(\omega_L,\omega_P)$, should be consistent with $penalty_{\tilde{h}} = \rho\, max\{\eta^{\min}_{\tilde{h}} - \text{cumulative net profit at hour } \tilde{h}, 0\}$. This logic results since at least one of the constraints (1m) or (1n) should be active at an optimal solution otherwise a contradiction with optimality since $penalty_{\tilde{h}}$ only appears in these two constraints but has a negative coefficient in the maximization problem.

3) _Bounds on Load Estimates_ The constraints (1o) and (1p) establish that the estimates for forward load must be between 0 and the maximum level allowed.

The next result shows that the optimization problem (1) that the retailer would need to solve always has a solution.

Theorem 2

The optimization problem (1) faced by the retailer always has a solution.

Proof

We will show that a solution to this problem exists by using Weirstrass' Theorem. Since the objective function is linear in its variables, it suffices to show that the feasible region given by (1b)-(1p) is non-empty and compact.

The closedness of the feasible region is guaranteed since the constraints are either linear or binary. To complete the demonstration of compactness, we show that each of the variables is bounded. Clearly, by definition the binary variables $y^1_{c,h}(\omega_L), y^2_{c,h}(\omega_L), y^3_{c,h}(\omega_L)$ are bounded. Theorem 1 showed that when the variables $x^1_{c,h}(\omega_L)$, $x^2_{c,h}(\omega_L), x^3_{c,h}(\omega_L)$ were non-zero, they were bounded as well. Constraints (1o) and (1p) show that the load estimate variables $LOAD^F_{c,e,h}$ are also bounded. The intermediate variables $\Delta_{c,h}(\omega_L), \Psi_{c,h}$ are also bounded in light of (10), (1p), (1b) and (1c). Lastly, we see that the penalty variables $penalty_{\tilde{h}}$ are bounded below by (1n). Assume for contradiction that there was a sequence of optimal values for the variable $penalty_{\tilde{h}}$

with $\tilde{h}$ fixed, that tended to $+\infty$. Clearly such a sequence could not be optimal since taking the maximum of the right-hand side of (12) and 0 would be feasible (relative to the penalty term) but would produce a higher function value in light of the negative coefficient for $penalty_{\tilde{h}}$ in the objective function. Since the right-hand side of (1m) involved terms that were bounded, this shows that this variable is bounded. Hence, the feasible region is compact.

As for the non-emptyness of the feasible region, consider the following feasible point.

- $LOAD^{F}_{c,e,h} = 0, \forall c, e, h$
- $\Delta_{c,h}(\omega_L) = -\sum_{e \in E(c)} N_e \left(LOAD^{A}_{e,h}(\omega_L)\right) = x^{1}_{c,h}(\omega_L) \le 0 \quad \forall c, h, \omega_L \in \Omega_L$
- $\Psi_{c,h} = 0, \forall c, h$
- $penalty_{\tilde{h}} = max\left\{\eta^{min}_{\tilde{h}} - \left[TOTPROF_{h_1 - 1}\right.\right.$

$$+ \sum_{h_1 \le h \le \tilde{h}} \sum_{c \in C} \sum_{e \in E(c)} PRICE^{EU}_{e,h} N_e LOAD^{A}_{e,h}(\omega_L) + \sum_{h_1 \le h \le \tilde{h}} \sum_{c \in C} \left\{ \gamma^{1}_{c,h} PRICE^{SM}_{h}(\omega_P) x^{1}_{c,h}(\omega_L) + \right.$$

$$\left.\left.\left. \gamma^{2}_{c,h} PRICE^{SM}_{h}(\omega_P) x^{2}_{c,h}(\omega_L) + \gamma^{3}_{c,h} PRICE^{SM}_{h}(\omega_P) x^{3}_{c,h}(\omega_L) \right\} - \sum_{h_1 \le h \le \tilde{h}} \sum_{c \in C} \sum_{e \in E(c)} PRICE^{SU}_{c,h} N_e LOAD^{F}_{c,e,h} \right], 0 \right\}$$

$\forall \tilde{h} \in \tilde{H}$

  - $x^{3}_{c,h}(\omega_L) = 0, \; y^{3}_{c,h}(\omega_L) = 0, \forall c, h, \omega_L \in \Omega_L$
  - $x^{2}_{c,h}(\omega_L) = 0, \forall c, h, \omega_L \in \Omega_L$

If $\sum_{e \in E(c)} N_e \left(LOAD^{A}_{e,h}(\omega_L)\right) > 0$ Then

$y^{1}_{c,h}(\omega_L) = 1, \; y^{2}_{c,h}(\omega_L) = 0, \forall c, h, \omega_L \in \Omega_L$

Else $\sum_{e \in E(c)} N_e \left(LOAD^A_{e,h}(\omega_L) \right) = 0$

$y^1_{c,h}(\omega_L) = 0$, $y^2_{c,h}(\omega_L) = 1$, $\forall c, h, \omega_L \in \Omega_L$

End If

■

6. Numerical Results

To test out the mixed integer linear program described above, we used PJM market data from (Gabriel *et al.*, 2002) but representing just two consecutive summer peak hours. In particular, we used the data points from June to August of 1998 referring to the peak periods of 7 am – 10 pm Monday to Friday. For purposes of simplicity we derived three point distributions for price and load based on these data where the three points represented "low", "medium" and "high" values; see Tables 2 and 3. "Low" values constituted the bottom quartile, "medium" values represented the medium two quartiles, and "high" values were the top quartile. Consistent with the findings and arguments discussed in (Gabriel *et al.*, 2002), the spot market prices and load were taken as independent of each other. [15] This is reasonable since the retailer load is specific to that retailer and may not reflect the conditions in the whole market as does the spot market price.

TABLE 2

Three-Point Distribution Used for Spot Market Prices

[15] In particular, two statistical tests were performed to confirm this hypothesis, one for the PJM market using 1488 hours in the spring of 2000 and one for the ERCOT market using 1272 hours in the summer and early fall of 2001. The first test revealed a correlation of –0.02975 between retailer load and spot market prices whereas the second test produced a value of 0.127446, confirming a negligible correlation in both cases.

LEVELS	$/MWh	PROBABILITY
Low	17.34	0.25
Medium	32.44	0.50
High	93.34	0.25

TABLE 3

Three-Point Distribution Used for Retailer Loads

LEVELS	MW	PROBABILITY
Low	530.81	0.25
Medium	652.59	0.50
High	799.28	0.25

Additionally, we considered two supply contracts c_1, c_2 with three end-user classes served as follows: $E(c_1) = \{e_1\}, E(c_2) = \{e_2, e_3\}$ and with just one end-user (N_e=1) in each class. Table 4 shows the default values for other key parameters that we used. Our point in presenting results for just two hours was validation that the model performed as expected on the small data set. In addition, this exercise provided direction on how to implement certain aspects of the model on a larger data set if desired as well as provided model sensitivities.

TABLE 4

Default Values for Parameters

PARAMETER	SYMBOL	VALUE
Penalty	ρ	10%
Tolerance cutoff value	$T_{c,h}$	8%
Maximum load forecast	$M_{c,h}$	1,000 MW
Min. acceptable cumulative profit	$\eta_{\tilde{h}}^{\min}$	$1,000
End-user class 1 prices	$PRICE_{1,h}^{EU}, h=1,2$	$15.198/ MWh
End-user class 2 prices	$PRICE_{2,h}^{EU}, h=1,2$	$16.014/ MWh
End-user class 3 prices	$PRICE_{3,h}^{EU}, h=1,2$	$16.170/ MWh
Supplier 1 contract price	$PRICE_{1,h}^{SU}, h=1,2$	$14.90/MWh
Supplier 2 contract price	$PRICE_{2,h}^{SU}, h=1,2$	$15.50/MWh
Hours when penalty applied	$\tilde{H}$	$\{1,2\}$
Total net profit before hour 1	$TOTPROF_{h_1-1}$	$15,000

The values shown in Tables 2, 3, and 4 were chosen to be reasonable as well as illuminate some of the various cases described in Table 1. To take into account that the prices (loads) were related from one hour to the next, we developed probability transition matrices as shown in Tables 5 and 6 using the Law of Total Probability (Clemen and Reilly, 2001) consistent with this notion from random walks. As an example, consider the probability of low prices in the second hour

$$P(L_2) = P(L_2/L_1)P(L_1) + P(L_2/M_1)P(M_1) + P(L_2/H_1)P(H_1) \tag{3}$$

where $L_h, M_h, H_h, h = 1,2$ are the events of low, medium, and high prices for hour *h*, respectively. The computations are similar for the other price levels and for the loads.

The values for the probabilities for low, medium, and high prices (loads) in hour 1 were obtained by using the total number of hours in each group divided by total number of hours considered for the summer peak (1040). The conditional probabilities $P(L_2 / L_1)$, etc. for price and load were calculated in a similar way. For example, from Table 5 we see that there was a probability of 0.70 that if the price was low in hour one that hour two's prices would also be low. The tendency of prices (loads) to stay in the same group from one hour to the next is demonstrated by noting that the transition matrices in Tables 5 and 6 are strictly diagonally dominant, i.e., the diagonal entries are larger than the sum of the absolute values of the entries in the rest of the row indicating the greater probability of staying at the same level. Additionally, these tables also show the rather unlikely event of transitioning directly from a low level to a high level.

TABLE 5

Price Transition Matrix

		Price Hr. 2		
		Low	Med	High
Price Hr.1	Low	0.700000	0.292308	0.007692
	Med	0.144231	0.740385	0.113462
	High	0.011538	0.223077	0.765385

TABLE 6

Load Transition Matrix

		Load Hr.2		
		Low	Med	High
Load Hr.1	Low	0.803846	0.196154	0.000000
	Med	0.095969	0.834933	0.067179
	High	0.000000	0.135135	0.864865

Given a starting set of values for the probabilities $P(L_1), P(M_1), P(H_1)$, one can obtain the associated probabilities in hour two by matrix multiplication using the transposes of the matrices from Tables 5 and 6 multiplied by the vector of values $P(L_1), P(M_1), P(H_1)$. Clearly this approach can be expanded in at least two ways for a time horizon larger than two hours. First, these conditional probabilities could involve more than just the previous hour's value. Thus, if desirable, one could produce the probability for the current hour's price (load) dependent on the previous *t* hours. Also, more than three levels for price (load) could be computed.

In the rest of this section we consider selected results when varying different values for parameters in the model formulation. In our numerical tests, we used the software package MPL (Maximal Software) and the solver XPRESS-MP (Dash Optimization) to solve the mixed integer programming problem specified above to obtain the results. Five different parameters were considered for this exercise as follows:

1. Probability of spot market price for three price levels, high, medium, and low (tests 1-5)
2. Maximum load forecast (MW) (test 6)

3. Penalty or Costs of Capital (%) (test 7)

4. Minimum acceptable cumulative profit ($) (test 8)

5. Tolerance cutoff value (%) (test 9)

Test Number 1

Test number 1 used the base case values for the spot market prices and probabilities from Table 4 and indicated a profit of $25,520 (for one hour), while showing that over estimating the load beyond the bandwidth tolerance (i.e., segment 3, the right-most in Figure 3) is best from the retailer's point of view. This results since even the minimum spot market price ($17.34) is greater than the end-user and supplier prices. We can observe that the result corresponds to case 1 from Table 1.

Test Number 2

For the other extreme, we considered spot market prices consistent with cases 5 or case 6 from Table 1 in which under-estimating the load is preferred. To simulate this test number 2, we set the spot market price equal to 1 with a price of 0.34$/MWh. As the retailer wants to maximize its profit, it will try to under-estimate the load as much as possible since the spot market price is lower than the supplier price and end-user prices. In effect, the most profitable strategy is to just buy all the electricity from the spot market and sell it to the end-users bypassing the suppliers. From Table 7 we see that the result is to estimate no load to the supplier with a net profit of $30,544 (corresponding to segment 1, the left-most in Figure 3).

Test Number 3

The next test was to set the spot market price equal to $15.8/MWh with a probability of 1. Based on the data that were used, cases 1, 2, 3, or 4 from Table 1 were appropriate. In explaining why theses cases were valid, it is necessary to summarize the relevant data. First, the prices for end-users 1,2, and 3 were respectively, $15.198, $16.014, $16.170 per MWh and the first end-user was covered by supplier 1 but the other two end-users were handled by supplier 2. The contract supply prices between the retailer and the suppliers were $14.90 and $15.50, respectively for these two suppliers. The closeness of the spot market prices, end-user prices, and supply prices makes for a variety of interesting cases relating to optimal retailer actions when one considers the three levels of load examined. In particular, for a low level of end-user load, optimally the retailer should over-estimate the load to both suppliers 1 and 2, corresponding to segment 3 in Figure 2 (cases 1 and 3 in Table 1, dependent on which supplier is being considered). For a medium level of end-user load, optimally the retailer should over-estimate the load to supplier 1 (segment 3) but should estimate the load within the 8% tolerance (if possible) relating to segment 2 in Figure 2. Lastly, for a high level of load, the retailer's optimal action is to over-estimate the load to supplier 1 (segment 3) but should under-estimate the load to supplier 2 (segment 1) corresponding to case 4 in Table 1. These results indicate the complexity of the optimal retailer decisions depending on the relative values of the various prices and the level of the load.

Test Number 4

Setting the spot market price to $93.34 with probability equal to 1 was the next test tried. This test corresponded to cases 1 or 2 but differed from the base case since only the high spot market price was allowed so segment 3 was chosen. The estimated load was always chosen to be the maximum possible (1000 MW) similar to the base case but the profit was larger due to the higher spot market price and the resulting higher settlement revenues to the retailer; segment 3 was selected.

Test Number 5

Test number 5 was to allow equal probabilities for the default spot market prices. Again we see that segment 3 was chosen for which forecasting the maximum of 1000 MW for both hours was optimal.

TABLE 7

Spot Market Price Sensitivity Analysis (Results for Each Hour)

	Test 1	Test 2	Test 3	Test 4	Test 5
Prob. Low Price	0.25	1	1	0	0.33
Prob. Med Price	0.5	0	0	0	0.33
Prob. High Price	0.25	0	0	1	0.33
Spot Price (Low)	$17.34	$0.34	$15.8	$17.34	$17.34
Spot Price (Med)	$32.44	$32.44	$32.44	$32.44	$32.44
Spot Price (High)	$93.34	$93.34	$93.34	$93.34	$93.34
Forward Load e1	1000	0	1000	1000	1000
Forward Load e2	1000	0	208.5	1000	1000
Forward Load e3	1000	0	1000	1000	1000
Profit	$25,520	$30,544	$1,575	$76,134	$28,938

These tests confirm that the model performs as expected but also indicate for this small data set that over-estimating the load can often be the best strategy for the retailer at least for the PJM data in question (this result is not always valid though, if one considers for example tests 2 and 3). This result of over-estimating coincides with one of the conclusions in (Gabriel *et al.*, 2002) in which always over estimating to the maximum level stochastically dominated other retailer strategies. [16]

Test Number 6

Another significant parameter besides the spot market price probability distribution was the maximum load forecast value; the default was 1000 MW. If this maximum value is low then the retailer has a limited quantity to over-estimate resulting in more purchasing (all things being equal) from the uncertain and volatile spot market price. This result is shown in Table 8. Also in this table, it is interesting to note that if the maximum load estimate increased by a factor of 10, the result profit did not change accordingly. At lower values in this table, the increase in profit was marginal. Only above 100 did a significant effect take place.

TABLE 8

Maximum Load Estimate Sensitivity Results (For Each Hour)

[16] After trying 12 different sets of probabilities for spot market prices, in which these probabilities were randomly generated, over-estimating the load was seen to be best relative to profit.

Max Forecast Estimate	0	10	100	1000	10000
Max Profit	-$55530	-$54673	-$46963	$25520	$585964
Estimated Supplier Load	0	10	100	1000	10000

Test Number 7

Another parameter whose values were adjusted was the penalty or cost of capital. When the penalty was increasing the profit decreased until the penalty overwhelmed the retailer's strategy. From that point on the retailer had no incentive to change the strategy. Figure 5 below depicts the profit as a function of this penalty parameter. Note that high values of the penalty were meant for illustrative purposes and would most likely not be in force in practice. It is interesting to note that from a penalty of less than 50% that segment 3 (over-estimating) was selected. Higher than this level, segment 2 (within tolerance) was chosen. One explanation for this is that when the penalty becomes prohibitive (50% or higher), the retailer will want to produce more accurate load estimates to minimize the down-side of the settlement risk since the penalty costs are already very high.

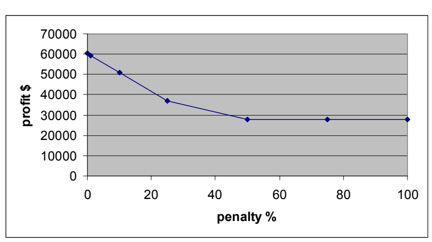

Fig. 5. Sensitivity of Profit to the Penalty Parameter.

Test Number 8

Another parameter that was varied was the minimum acceptable cumulative profit value. When this parameter was increasing, the profit decreased since this parameter will increase the cost of the penalty. This behavior, using the values $0, $100, $1000, $10,000, $100,000, $500,000, and $1,000,000 is shown in Figure 6.

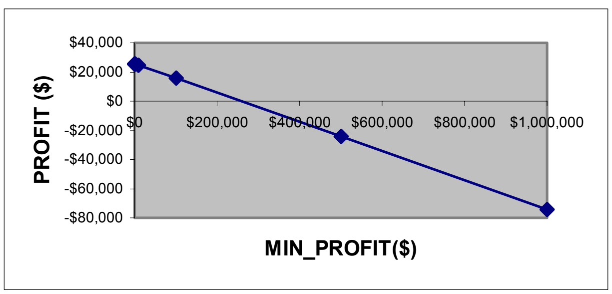

Fig. 6. Sensitivity of Profit to the Minimum Cumulative Profit Parameter.

Test Number 9

The last sensitivity analysis concerned the tolerance cutoff value. Increasing the tolerance allows for the retailer to more of the time split the settlement profits or losses with the supplier. From Figure 7 it appears that increasing this parameter is less beneficial to the retailer. The rationale is that for these PJM market data, on average the retailer is probably making more money in the spot market (consistent with over-estimating the load) and that increasing the tolerance diminishes this source of revenue since it is split with the supplier. This is important in contractual arrangements since all things being equal, the retailer may want to strive for a lower tolerance, a somewhat counter-intuitive result considering that the retailer would more of the time be over- or under-estimating the load.

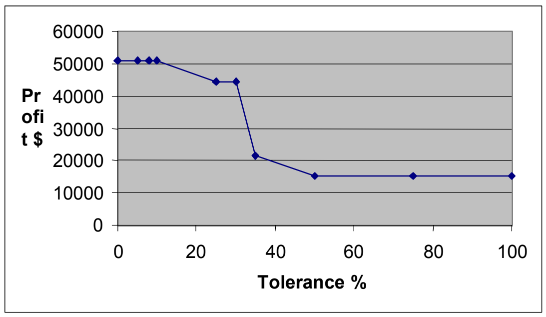

Fig. 7. Sensitivity of Profit for Both Hours to the Bandwidth Tolerance Cutoff Value.

7. Conclusions

In this paper we have presented a mixed integer linear program to optimize the expected net revenue for power retailers. This model takes into account that the spot market prices and actual retailer loads to their end-users are uncertain and can be described by appropriate probability distributions. We showed that the logic of the model reflected various important contractual items and we showed that this model always has a solution. A small example of two hours was also presented to clarify the actual usage of the model as well as provide some sensitivity analyses.

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